Searching for Supersymmetric Dark Matter- The Directional Rate for Caustic Rings.

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Abstract. The detection of the theoretically expected dark matter is central to particle physics and cosmology. Current fashionable supersymmetric models provide a natural dark matter candidate which is the lightest supersymmetric particle (LSP). The theoretically obtained event rates are usually very low or even undetectable. So the experimentalists would like to exploit special signatures like the directional rates and the modulation effect. In the present paper we study these signatures focusing on a specific class of non-isothermal models involving flows of caustic rings.

1 Introduction

In recent years the consideration of exotic dark matter has become necessary in order to close the Universe [1]. Recent data from the High-z Supernova Search Team [2] and the Supernova Cosmology Project [3] , [4] suggest the presence of a cosmological connstanta Λ. In fact the situation can be adequately described by a barionic component $\Omega_B = 0.1$ along with $\Omega_{CDM} = 0.3$ and $\Omega_\Lambda = 0.6$ (see also Turner, these proceedings).

Since this particle is expected to be very massive, $m_\chi \geq 30GeV$, and extremely non relativistic with average kinetic energy $T \leq 100KeV$, it can be directly detected [5]$^-$[6] mainly via the recoiling nucleus.

Using an effective supersymmetric Lagrangian at the quark level, see e.g. Jungman et al [1] and references therein , a quark model for the nucleon [7,10] and nuclear wave functions [6] one can obtain the needed detection rates. They are typically very low. So experimentally one would like to exploit the modulation of the event rates due to the earth's revolution around the sun. In our previous work [8], [9] we found enhanced modulation, if one uses appropriate asymmetric velocity distribution. The isolated galaxies are, however, surrounded by cold dark matter , which, due to gravity, keeps falling continuously on them from all directions [11]. It is the purpose of our present paper to exploit the results of such a scenario.

2 The Basic Ingredients for LSP Nucleus Scattering

The differential cross section can be cast in the form [9]:

$$d\sigma(u,\upsilon) = \frac{du}{2(\mu_r b\upsilon)^2}[(\bar{\Sigma}_S + \bar{\Sigma}_V \ \frac{\upsilon^2}{c^2}) \ F^2(u) + \bar{\Sigma}_{spin} F_{11}(u)] \tag{1}$$

$$\bar{\Sigma}_S = \sigma_0 (\frac{\mu_r}{m_N})^2 \{A^2 [(f_S^0 - f_S^1 \frac{A-2Z}{A})^2] \simeq \sigma_{p,\chi^0}^S A^2 (\frac{\mu_r}{m_N})^2 \tag{2}$$

$$\bar{\Sigma}_{spin} = \sigma_{p,\chi^0}^{spin} \ \zeta_{spin} \tag{3}$$

$$\zeta_{spin} = \frac{(\mu_r/m_N)^2}{3(1+\frac{f_A^0}{f_A^1})^2} [(\frac{f_A^0}{f_A^1} \Omega_0(0))^2 \frac{F_{00}(u)}{F_{11}(u)} + 2\frac{f_A^0}{f_A^1} \Omega_0(0)\Omega_1(0) \frac{F_{01}(u)}{F_{11}(u)} + \Omega_1(0))^2] \tag{4}$$

$$\bar{\Sigma}_V = \sigma_{p,\chi^0}^V \ \zeta_V \tag{5}$$

$$\zeta_V = \frac{(\mu_r/m_N)^2}{(1+\frac{f_V^1}{f_V^0})^2} A^2 \, (1 - \frac{f_V^1}{f_V^0} \ \frac{A-2Z}{A})^2 [(\frac{\upsilon_0}{c})^2 [1 - \frac{1}{(2\mu_r b)^2} \frac{2\eta+1}{(1+\eta)^2} \frac{\langle \ 2u \ \rangle}{\langle \ \upsilon^2 \ \rangle}] \tag{6}$$

σ_{p,χ^0}^i = proton cross-section with $i = S, spin, V$ given by:
$\sigma_{p,\chi^0}^S = \sigma_0 \ (f_S^0)^2$ (scalar) , (the isovector scalar is negligible, i.e. $\sigma_p^S = \sigma_n^S$)
$\sigma_{p,\chi^0}^{spin} = \sigma_0 \ \ 3 \ (f_A^0 + f_A^1)^2$ (spin) , $\sigma_{p,\chi^0}^V = \sigma_0 \ (f_V^0 + f_V^1)^2$ (vector)
where m_p is the proton mass, $\eta = m_x/m_N A$, and μ_r is the reduced mass and

$$\sigma_0 = \frac{1}{2\pi} (G_F m_N)^2 \simeq 0.77 \times 10^{-38} cm^2 \tag{7}$$

$$u = q^2 b^2/2 \ \ or \ \ Q = Q_0 u, \qquad Q_0 = \frac{1}{A m_N b^2} \tag{8}$$

where b is (the harmonic oscillator) size parameter, q (Q) is the momentum (energy) transfer to the nucleus. In the above expressions $F(u)$ is the nuclear form factor and $F_{\rho\rho'}(u)$ are the spin form factors [6] (ρ, ρ' are isospin indices) The differential non-directional rate can be written as

$$dR = dR_{non-dir} = \frac{\rho(0)}{m_\chi} \frac{m}{A m_N} d\sigma(u, \upsilon) |\upsilon| \tag{9}$$

where $\rho(0) = 0.3 GeV/cm^3$ is the LSP density in our vicinity and m is the detector mass

The directional differential rate [12] in the direction $\hat{e}$ is given by :

$$dR_{dir} = \frac{\rho(0)}{m_\chi} \frac{m}{A m_N} \upsilon.\hat{e} H(\upsilon.\hat{e}) \ \frac{1}{2\pi} \ d\sigma(u, \upsilon) \tag{10}$$

where H the Heaviside step function. The factor of $1/2\pi$ is introduced, since we have chosen to normalize our results to the usual differential rate.

We will now examine the consequences of the earth's revolution around the sun (the effect of its rotation around its axis is expected to be negligible) i.e. the modulation effect.

Following Sikivie we will consider $2 \times N$ caustic rings, (i,n) , i=(+.-) and n=1,2,...N (N=20 in the model of Sikivie et al), each of which contributes to the local density a fraction $\bar{\rho}_n$ of the the summed density $\bar{\rho}$ of each type $i = +, -$. and has velocity $\mathbf{y}_n = (y_{nx}, y_{ny}, y_{nz})$, in units of $\upsilon_0 = 220 \ Km/s$, with respect to the galactic center.

We find it convenient to choose the z-axis in the direction of the motion of the the sun, the y-axis is normal to the plane of the galaxy and the x-axis is in the radial direction. The needed quantities are taken from the work of Sikivie (table 1 of last Ref. [11]) by the definitions $y_n = \upsilon_n/\upsilon_0, y_{nz} = \upsilon_{n\phi}/\upsilon_0, y_{nx} = \upsilon_{nr}/\upsilon_0, y_{ny} = \upsilon_{nz}/\upsilon_0$. This leads to a velocity distribution of the form:

$$f(\upsilon') = \sum_{n=1}^{N} \delta(\upsilon^{'} - \upsilon_0 \, \mathbf{y}_n) \tag{11}$$

The velocity of the earth around the sun is given by [6].

$$\upsilon_E \ = \ \upsilon_0 \ + \ \upsilon_1 \ = \ \upsilon_0 + \upsilon_1(\, sin\alpha \, \hat{\mathbf{x}} - cos\alpha \, cos\gamma \, \hat{\mathbf{y}} + cos\alpha \, sin\gamma \, \hat{\mathbf{z}} \,) \tag{12}$$

where α is the phase of the earth's orbital motion, $\alpha = 0$ around second of June. In the laboratory frame we have [9] $\upsilon = \upsilon^{'} \ - \ \upsilon_E$

3 Event Rates

Integrating Eq. (9) we obtain for the total non-directional rate

$$R = \bar{R} \, t \, \frac{2\bar{\rho}}{\rho(0)} [1 - h(a, Q_{min}) cos\alpha] \tag{13}$$

The integration was performed from$u = m_{min}$ to $u = u_{max}$, where

$$u_{min} = \frac{Q_{min}}{Q_0}, u_{max} = min(\frac{y^2_{esc}}{a^2}, max(\frac{y^2_n}{a^2}) \ , \ n = 1, 2, ..., N) \tag{14}$$

Here $y_{esc} = \frac{\upsilon_{esc}}{\upsilon_0}$, with $\upsilon_{escape} = 625 Km/s$ is the escape velocity from the galaxy. Q_{min} is the energy transfer cutoff imposed by the detector and $a = [\sqrt{2} \mu_r b \upsilon_0]^{-1}$. Also $\rho_n = d_n/\bar{\rho}, \bar{\rho} = \sum_{n=1}^{N} \ d_n$ (for each flow +,-). In the Sikivie model [11] $2\bar{\rho}/\rho(0) = 1.25$. $\bar{R}$ is obtained [5] by neglecting the folding with the LSP velocity and the momentum transfer dependence, i.e. by

$$\bar{R} = \frac{\rho(0)}{m_\chi} \frac{m}{A m_N} \sqrt{\langle \upsilon^2 \rangle} [\bar{\Sigma}_S + \bar{\Sigma}_{spin} + \frac{\langle \upsilon^2 \rangle}{c^2} \bar{\Sigma}_V] \tag{15}$$

and it contains all SUSY parameters except m_χ The modulation is described in terms of the parameter h. The effect of folding with LSP velocity and the nuclear form factor is taken into account by t (see table 1)

There are now experiments under way aiming at measuring directional rates , i.e. the case in which the nucleus is observed in a certain direction. The rate will depend on the direction of observation, showing a strong correlation with the direction of both the sun's and the earth's motion. In the favorable situation the rate will merely be suppressed by about a factor of 2π relative to the non-directional rate. This is due to the fact that one does not now integrate over the azimuthal angle of the nuclear recoiling momentum.

Table 1. The quantities t and h entering the total non-directional rate in the case of the target ${}_{53}I^{127}$ for various LSP masses and Q_{min} in KeV. Also shown are the quantities r^i_j, h^i_j $i=u,d$ and $j=x,y,z,c,s$, entering the directional rate for no energy cutoff. For definitions see text.

		LSP mass in GeV						
Quantity	Q_{min}	10	30	50	80	100	125	250
t	0.0	1.451	1.072	0.751	0.477	0.379	0.303	0.173
h	0.0	0.022	0.023	0.024	0.025	0.026	0.026	0.026
r^u_z	0.0	0.726	0.737	0.747	0.757	0.760	0.761	0.761
r^u_y	0.0	0.246	0.231	0.219	0.211	0.209	0.208	0.208
r^u_x	0.0	0.335	0.351	0.366	0.377	0.380	0.381	0.381
h^u_z	0.0	0.026	0.027	0.028	0.029	0.029	0.030	0.030
h^u_y	0.0	0.021	0.021	0.020	0.020	0.019	0.019	0.019
h^u_x	0.0	0.041	0.044	0.046	0.048	0.048	0.049	0.049
h^u_c	0.0	0.036	0.038	0.040	0.041	0.042	0.042	0.042
h^u_s	0.0	0.036	0.024	0.024	0.023	0.023	0.022	0.022
r^d_z	0.0	0.274	0.263	0.253	0.243	0.240	0.239	0.239
r^d_y	0.0	0.019	0.011	0.008	0.007	0.007	0.007	0.007
r^d_x	0.0	0.245	0.243	0.236	0.227	0.225	0.223	0.223
h^d_z	0.0	0.004	0.004	0.004	0.004	0.004	0.004	0.004
h^d_y	0.0	0.001	0.000	0.000	0.000	0.000	0.000	0.000
h^d_x	0.0	0.022	0.021	0.021	0.020	0.020	0.020	0.020
h^d_c	0.0	0.019	0.018	0.018	0.017	0.017	0.017	0.017
h^d_s	0.0	0.001	0.001	0.000	0.000	0.000	0.000	0.000
t	10.0	0.000	0.226	0.356	0.265	0.224	0.172	0.098
h	10.0	0.000	0.013	0.023	0.025	0.025	0.026	0.026
t	20.0	0.000	0.013	0.126	0.139	0.116	0.095	0.054
h	20.0	0.000	0.005	0.017	0.024	0.025	0.026	0.026

We need distinguish the following cases: a) $\hat{e}$ has a component in the sun's direction of motion, i.e. $0 < \theta < \pi/2$, labeled by i=u (up). b) Detection in the opposite direction, $\pi/2 < \theta < \pi$, labeled by i=d (down). Thus we find :

1. In the first quadrant (azimuthal angle $0 \leq \phi \leq \pi/2$).

$$R^i_{dir} = \bar{R}\frac{2\bar{\rho}}{\rho(0)}\frac{t}{2\pi}[(r^i_z - \cos\alpha\ h^i_1)\mathbf{e}_z.\mathbf{e}$$
$$+ (r^i_y + cos\alpha h^i_2 + \frac{h^i_c}{2}(|cos\alpha| + cos\alpha))|\mathbf{e}_y.\mathbf{e}|$$
$$+ (r^i_x - sin\alpha h^i_3 + \frac{h^i_s}{2}(|sin\alpha| - sin\alpha))|\mathbf{e}_x.\mathbf{e}|] \tag{16}$$

2. In the second quadrant (azimuthal angle $\pi/2 \leq \phi \leq \pi$)

$$R^i_{dir} = \bar{R}\frac{2\bar{\rho}}{\rho(0)}\frac{t}{2\pi}[(r^i_z - \cos\alpha\ h^i_1)\mathbf{e}_z.\mathbf{e}$$
$$+ (r^i_y + cos\alpha h^i_2(u) + \frac{h^i_c}{2}(|cos\alpha| - cos\alpha))|\mathbf{e}_y.\mathbf{e}|$$
$$+ (r^i_x + sin\alpha h^i_3 + \frac{h^i_s}{2}(|sin\alpha| + sin\alpha))|\mathbf{e}_x.\mathbf{e}|] \tag{17}$$

3. In the third quadrant (azimuthal angle $\pi \leq \phi \leq 3\pi/2$).

$$R^i_{dir} = \bar{R}\frac{2\bar{\rho}}{\rho(0)}\frac{t}{2\pi}[(r^i_z - \cos\alpha\ h^i_1)\mathbf{e}_z.\mathbf{e}$$
$$+ (r^i_y - cos\alpha h^i_2(u) + \frac{h^i_c(u)}{2}(|cos\alpha| - cos\alpha))|\mathbf{e}_y.\mathbf{e}|$$
$$+ (r^i_x + sin\alpha H^i_3 + \frac{h^i_s}{2}(|sin\alpha| + sin\alpha))|\mathbf{e}_x.\mathbf{e}|] \tag{18}$$

4. In the fourth quadrant (azimuthal angle $3\pi/2 \leq \phi \leq 2\pi$)

$$R^i_{dir} = \bar{R}\frac{2\bar{\rho}}{\rho(0)}\frac{t}{2\pi}[(r^i_z - \cos\alpha\ h^i_1)\mathbf{e}_z.\mathbf{e}$$
$$+ (r^i_y - cos\alpha h^i_2 + \frac{h^i_c}{2}(|cos\alpha| - cos\alpha))|\mathbf{e}_y.\mathbf{e}|$$
$$+ (r^i_x - sin\alpha h^i_3 + \frac{h^i_s}{2}(|sin\alpha| - sin\alpha))|\mathbf{e}_x.\mathbf{e}|] \tag{19}$$

4 Conclusions

We have calculated the parameters describing characteristic signatures needed to reduce the formidable backgrounds in the direct detection of SUSY dark matter, such as : a) The modulation effect, correlating the rates with the motion of the Earth and b) The directional rates, correlated with both with the velocity of the sun and that of the Earth (see table 1).

We have focused on the LSP density and velocity spectrum obtained from a recently proposed non-isothermal model, involving caustic rings [11]. Our results for isothermal models have appeared elsewhere [8,9].

The quantities t and h are given in table 1. We see that the maximum in this model does not occur around June 2nd, but about six months later. The difference between the maximum and the minimum is about 4%, i.e. smaller than that predicted by the asymmetric isothermal models [8,9].

For the directional experiments we found that the biggest rates are obtained close to the direction of the sun's motion. They are suppressed compared to the usual non-directional rates by the factor $f_{red} = \kappa/(2\pi)$, $\kappa = u_z^i$. We find $\kappa \simeq 0.7$, $i = up$ (observation in the sun's direction of motion) while $\kappa \simeq 0.3$, $i = down$ (in the opposite direction). The modulation is a bit larger than in the non-directional case. The largest difference between the maximum and the minimum, 8%, occurs not the sun's direction of motion, but in the x-direction (galactocentric direction).

In the case of the isothermal models the reduction factor along the sun's direction of motion is now given $f_{red} = t_0/(4\pi\ t) = \kappa/(2\pi)$. Using the values of t_0 obtained previously [9], we find that κ is around 0.6 for the symmetric case and around 0.7 for maximum asymmetry ($\lambda = 1.0$). The modulation of the directional rate depends on the direction of observation. It is generally larger and increases with the asymmetry parameter λ. For $Q_{min} = 0$ it can reach values up to 23%. Values up to 35% are possible for large Q_{min}, but at the expense of the total number of counts [9].

Finally in all cases t deviates from unity for large reduced mass. Thus when extracting the LSP-nucleon cross section from the data one must divide by t.

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